

What is the value of $\infty - \infty$?

Is it 0? Is it ∞ ? WHAT???

Think back to the Hilbert's Hotel problem:

Imagine that I have an infinite number of hotel rooms, each numbered 1, 2, 3, 4, ...

Then I give you all of them. I would have none left, so $\infty - \infty = 0$

On the other hand, if I give you all of the odd-numbered ones, then I still have an infinite number left. So $\infty - \infty = \infty$

Now suppose that I give you all of them except for the first seven. Then $\infty - \infty = 7$

While this doesn't explain *why* this is indeterminate, hopefully you can agree that it is indeterminate!

Another argument that $\infty - \infty$ is indeterminate...

Suppose we start with the assumption that $\infty - \infty = 0$

$\infty - \infty = 0$	$\infty - \infty = 0$	$\infty - \infty = 0$
$\infty - \infty + 1 = 0 + 1$	$\infty - \infty + \infty = 0 + \infty$	Since $\infty = \infty + \infty \dots$
$(\infty + 1) - \infty = 0 + 1$	$(\infty + \infty) - \infty = 0 + \infty$	$(\infty + \infty) - \infty = 0$
$\infty - \infty = 1$	Since $\infty = \infty + \infty \dots$	$\infty + (\infty - \infty) = 0$
	$\infty - \infty = \infty$	$\infty = 0$