

Summary of the convergence tests that may appear on the AP Calculus BC exam.

Test Name	The series ...	will converge if	Or will diverge if	Comments
n^{th} -term test	$\sum_{n=1}^{\infty} a_n$		$\lim_{n \rightarrow \infty} a_n \neq 0$	For divergence only; the converse is false.
Geometric series	$\sum_{n=1}^{\infty} ar^{n-1}$	$-1 < r < 1$	$r \leq -1$ or $r \geq 1$	Sum = $\frac{a}{1-r}$
Alternating series test	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$ a_{n+1} < a_n $ and $\lim_{n \rightarrow \infty} a_n = 0$		Error bound $ S_{\infty} - S_n < a_{n+1} $
Integral test	$\sum_{n=1}^{\infty} a_n$ and $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x) dx$ converges	$\int_1^{\infty} f(x) dx$ diverges	f must be continuous, positive and decreasing.
p -series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Direct comparison test	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Ratio test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \frac{ a_{n+1} }{ a_n } < 1$	$\lim_{n \rightarrow \infty} \frac{ a_{n+1} }{ a_n } > 1$	If $\lim_{n \rightarrow \infty} \frac{ a_{n+1} }{ a_n } = 1$ the ratio test cannot be used.

Other useful convergence tests that may be used.

Test Name	The series ...	will converge if	Or will diverge if	Comments
Limit comparison test	$\sum_{n=1}^{\infty} a_n$	$a_n > 0, b_n > 0$ $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$a_n > 0, b_n > 0$ $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Root test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} < 1$	$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} > 1$	The test cannot be used if $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = 1$