The AP Calculus Exam

How, not only to Survive, but to Prevail…

By Lin McMullin

The AP Calculus exam is the culmination of all of the years you've spent in high school studying mathematics. It's all led up to this. The calculus you study in the last year completes the prior years of preparation. If you are reading this at the beginning of the year keep these things in mind as you go through the year. If you are reading this only a few weeks before the test think back and see how these things fit together.

Everything in calculus, and mathematics in general, is best understood verbally, numerically, analytically (that is, through the use of equations and symbols) and graphically. Look at everything from these four perspectives. Look at the relationships among them — how the same idea shows up in words, in equations, in numbers and in graphs.

For example: numerically a linear function is one which when written as a table of values, regular changes in the x-values produce regular changes in the y-values. Graphically a linear function has a graph that is a straight line. Analytically it is one whose equation can be written as $y = mx + b$. And the three ways are interrelated: The ratio of the changes in the table is the number m in the equation; the graph can be drawn using the number m by going up and over from one point to the next. The idea of the slope as "rise over run" expresses this verbally. Everything in mathematics and in the calculus works that way.

- Learn the concepts — the exam emphasizes concepts
- Learn the procedures and formulae — even though the concepts are more important than the computations you still have to do computations. Like it or not, learn to do the algebra, the arithmetic and the graphs.
- Learn to be methodical — work neatly and carefully all year.

- Think about what you are doing. Watch yourself work. It is natural to concentrate on the material you know and can do, but you need to concentrate on the things you do not (yet) know how to do. You can learn much from your mistakes.
- Learn from your mistakes. Look at a wrong answer as a green light to go in that direction until you've reached the right answer.

Reviewing for the Exam

In the few weeks before the AP Exam you will need to review what you have studied, firm up what you have learned, work on your areas of weakness and yes, memorize some formulas. You also need to prepare for the exam itself by learning what kinds of questions will be asked and how to best answer them. Specifically

- Understand the format of the exams. (See below). Know how your knowledge will be tested.
- STUDY WHAT YOU DO NOT KNOW. That may seem obvious but many people enjoy getting the right answers so much that they only review the stuff they know. The time to concentrate on what you know is when you are taking the test.
- Practice writing free-response answers. The College Board publishes copies of student answer from past years. If your teacher has some of these, look at them and learn what is expected and what is not needed.
- Plan your review carefully. Don't try to cram the weekend before the exam. The day before the test: relax, get psyched, and get a good night's sleep. The day of the test eat a good breakfast. The test is grueling, even though you're up for it. Bring a snack for the brief break between the multiple-chose and free-response sections.

Calculators

The reason calculators are so important in learning mathematics is that they allow you do the graphical and numerical work easily, quickly and accurately. You should use your calculator all year, on homework, tests and when studying. Learn how to use it efficiently. Learn its strengths and weaknesses.

You may use your calculator any way you wish. There are four types of things you should definitely know how to do. They are

- Plot the graph of a function within an arbitrary viewing window,
- Solve equations numerically. One way to do this is to graph both sides of the equation and find the point(s) of intersection,
- Calculate numerically the value of the derivative of a function at a point, and
- Calculate numerically the value of a definite integral.

You may have programs in your calculator; but you will not be asked to use them. The questions on the exam are designed so that someone with a program, or a more expensive calculator or a computer algebra system, has no advantage over someone who does not. This includes many of the built-in programs.

Be sure your calculator is set in Radian mode.

Numerical answers may be left unsimplified and in terms of π, e, etc. There is no reason to change an answer to a decimal if you don't have to. (Why take the chance on pushing the wrong button?)

Install fresh batteries before the exam.

The Format of the Exams

There are two parts to the AP exams: a multiple-choice section and a free-response section. The number of questions and timing may change slightly from year to year. Be sure you check the current College Board publications for your exam.

Both sections count equally towards your final grade. Both sections cover

the full range of topics. It is natural to expect that different classes will cover some topics in greater detail than others; the exam will evaluate your knowledge of the calculus. It is *not* necessary to answer all the questions to get a good score. In fact the exam is made so that the average score will be about 50%; this is usually a score of three.

The Current AP Calculus Exam format is

Section I Part A (60 minutes) 30 multiple-choice questions for which you may **not** use a calculator. Each question has four choices. Answers are put on a mark sense (bubble) sheet and machine scored. There is no penalty for unanswered questions or incorrect answer.

Section I Part B (45 minutes) 15 multiple-choice questions. Each question has four choices. You may use your calculator on this section. Some of these questions, less than half, require the use of a graphing calculator, the others do not. There is no penalty for unanswered questions or incorrect answer.

Section II Part A (30 minutes) Two Free-Response questions. In this section you will find longer questions with several related parts. You will need your calculator for some parts (but not all parts) of questions in this section. You are required to show your work in this section. You may continue work on this section *without* a calculator after you start part B.

Section II Part B (60 minutes) Four Free-Response questions. You may **not** use your calculator on this section. In this section you will find longer questions with several related parts. You are required to show your work in this section. You may use part of this time to work on Section II, Part A *without* a calculator

Sections I and II count equally towards your final score.

Multiple Choice Questions

Read each question carefully and look at the answer choices. Do the ones you are sure of. Don't struggle over one that isn't working out. Remember your time is limited and you do not need to answer all of the questions. There is no penalty for guessing, nevertheless, don't guess blindly. Try to eliminate one or more of the choices before guessing.

Do not waste time on questions you are struggling with. A good procedure is to do all the "easy" questions, the ones you are sure of, first. Then go back and do the more difficult ones you think you can do, and then, if there is time, try the ones you are not sure you can do. Do not leave any question unanswered.

Be sure to bubble your answer in the correct space on the answer sheet.

Types of Multiple Choice Questions

- One type of question may ask for a computation (a limit, a derivative, a definite or indefinite integral) and give five possible answers: be aware that answers which result from predictable mistakes are among the choices — work carefully, just because your answer is there doesn't mean it's correct.
- Another type may ask you only to set up a problem: glancing at the answer choices before you start may keep you from doing too much work.
- Some questions ask you to choose the one true or one false statement from a list of five statements: be sure you know if you are looking for a true or a false statement.
- Another type of question asks which of three statements is true (or false): the answer may be any one or some combination of the statements.
- Another type may ask you to choose the correct table or graph from among five choices.
    - "None of the above" is never a choice.

Free-response Questions

The general directions for Section II require you to show your work and indicate the methods you use to arrive at your answers. In addition, parts of questions may say, "Justify your answer" or "Explain your reasoning" or "Show the analysis that leads to your conclusion." Your answers will be read by calculus teachers who will judge your work. It is important that you clearly show how you arrived at your answer. Unsupported answers lose points even if the final answer is correct.

The questions are designed to show the breadth and depth of your knowledge. There are some common types of questions that are asked. There will also be questions asked in new and original ways.

Some things to keep in mind about free-response questions:

- Don’t write a long essay: it's not necessary. Do show the work that you do, so that the reader will understand you. You may use common terms and names like “the first derivative test.” You do not need to name theorems. Explain in words and symbols what you see in the given information (the graph or table) that leads you to your conclusion; relate your reasoning to the given information or graph.
- Your justifications and explanations must be in word and symbols. Number lines are an excellent way to organize the information, but they do not count as justifications. Readers are forbidden to even look at number line justifications.
- The free-response section of the exam rarely requires long complicated computation; if you find yourself doing a long complicated computation you’ve probable gone wrong somewhere and should start over.
- Do not explain how to do the problem you cannot do. A general explanation without work will receive no credit. You must do the problem you are given.
- *Avoid simplifying numerical answers*. If you get 1 + 1 for your answer, leave it that way. Answers may be left unsimplified as fractions, radicals, powers of e, in terms of π, etc. Do not take a chance of pushing the wrong button once you have an acceptable answer. If you do arithmetic it must be done correctly. Every year students find the correct answer, simplify it or change it to a decimal incorrectly and lose a point.
- Decimal answers should only be given for computation you had to do on your calculator. Decimal answers (for example a definite integral on a calculator) are acceptable even if an exact answer is possible.
- If you make a mistake cross it out. Crossed out work is not read or graded. If you leave wrong work on your paper (not crossed out) it will be read and may affect your score.
- If you work the problem two different ways, choose the best one and put an X through the other. If both are left, they will both be scored and the scores will

be averaged and rounded down. This can lower your score even if one solution is perfect.

- Standard notation must be used. Don't use calculator notation. (For example: `fnInt(x2,x,Ø,2)` is not acceptable, use the standard $\int_0^2 x^2\,dx$.

- Answers without work may not receive full credit. Don't do work on a calculator without indicating what you are doing. Specifically, when using a calculator:
  - If you are solving an equation, write the equation on your paper and put the solution with it.
  - If you are computing a numerical derivative, write the function whose derivative you are computing. For example write $f'(3.4)$; you do not have to write the derivative itself.
  - If you are evaluating a definite integral write the integral on your paper and put the calculator answer next to it; you do not need to show the work in between (the antiderivative).

- Different calculators have different built-in utilities (for example the ability to find points of inflection, or maximum values of a function). You may have programs in your calculator to do things such as the left- or right-Riemann sum (LRAM, RRAM) of the Trapezoidal approximation. However, if you use such a built-in utility or a special program to do something other than the four things listed previously, you must show the complete set-up (the terms of the or Riemann sum approximation or the Trapezoidal approximation, the computation and analysis of the second derivative required to find a point of inflection etc.) on your paper. Only the four things listed above may be done without further explanation.

- Don't put things where they are not needed. Work must be shown on the part of the answer booklet where it is used. For example, if you need a derivative in part (b) of a question and you have it in part (a) where it is not needed, you

will not get credit for finding the derivative (in either part). Either copy it in part (b) or draw an arrow over to where you wrote it. You must show you know where you need the derivative as well as your ability to find it. Likewise, do not put work on the graph or drawing. It may not be read unless you specifically refer to it in the part of the answer booklet where you used it.

- The parts of a free-response question are related to each other. This can help you in two ways:
  - Most of the time each part may be answered without reference to the other parts. Read and try of all the parts: if you cannot do part (a) maybe you can do part (b). Perhaps doing part (b) will give you a hint on how to do part (a).
  - Other times the one part will lead to the next: this is done to help you find your way through the problem. Keep in mind that this may be the case and work your way from part (a) to part (b) to part (c) even if you're not sure where the problem is heading.
- If you have the wrong answer in one part of a question and use it correctly in another part, you will not lose credit twice. Unless the wrong answer greatly simplifies the next part, the second answer will be read as if your first answer were correct.
- Try all of the free-response questions. They are not arranged in order of difficulty. They are written so that the first parts are easier in order to help you get started. Even if you don't get the entire problem, earning some points are better than earning no points.

Decimals and Arithmetic

Some answers, the evaluation of definite integrals is an example, must be written as decimals because they are found using a graphing calculator. These answers, and other answers that you choose to change to decimals, must be correct to three places past the decimal point. This means that the answer may be rounded to three decimal places, truncated after the third decimal place or left with more than three decimal places as long

as the first three are correct. An answer of π, which should be left as π, may be given as 3.1415926535898…, 3.142, 3.141, 3.1416785 or even 3.142768. If the number ends in zeros, they may be omitted; thus 17.320 may be given as 17.32 and 56.000 may be given as 56.

Decimals answers are not required. Too often, students may choose to give decimal answers when they are not required. Once a free-response answer is entirely in terms of numbers there is no need to change the number to a decimal. For example, something like $-\frac{1}{2}\cos(4)+\frac{7}{2}$ is sufficient. A Riemann sum answer such as $\left(\frac{1}{2}\right)(3-2)+(4)(5-3)+(0.25)(13-8)$ is acceptable. Once your answer has no variable, you are done – go onto the next part of the question.

If the decimal is correct (to three decimal places) then you will receive the credit. However, if you change a correct answer to an incorrect decimal (including one with too few decimals) then you will lose credit.

Rounding too soon is another common mistake made by students. Computations should be done with more decimal places than is required in the final answer. Learn how to store the intermediate values in your calculator and recall them when you need them in a computation. If premature rounding affects the three decimal place accuracy of the final answer, you will not be given the answer point. However, a rounded answer used in the next part of a problem will not be held against you.

The moral is: avoid arithmetic, avoid decimals, and avoid rounding. Do not take a chance of changing a correct answer to an incorrect answer.

Common Free-Response Mistakes

- Algebra and arithmetic mistakes – remember simplifying algebraic expression and doing arithmetic are *not* required.
- Missing limits of integration.
- Not considering the end points of an interval (for example, when looking for the absolute maximum value of a function).

- Giving answers from points outside the given interval.
- Not giving both coordinates of a point when required.
- Giving both coordinates when only one is asked for. Remember "value of a function" means the *y*-value and "maximum value of a function" means the absolute maximum *y*-value.
- Not having your calculator in radian mode.
- Not answering the question that was asked even though all the work is correct. If it is a yes or no question, say "yes" or "no."
- Ignoring units of measure when they are specifically asked for.
- Family of function problems: Questions that start with a phrase like, "This question deals with functions defined by $f(x) = 1 + b\sin(x)$ where *b* is a positive constant..." are meant to be done in general, *not* for a specific value of *b.* Even if you get the correct answer using a specific value of *b*, you may lose points. The reason is that, because you used a particular value, you have no way to be sure that your answers are true for all values of *b.*
- Curve Fitting: There will be a function given as a graph or a table of values with no equation. You are being asked to demonstrate that you can work from the graphical or numerical data. The questions that follow can be answered without an equation. You may have learned to approximate functions using various curve fitting (regression) operations built into your calculator. *This should be avoided.* While this is a perfectly good approach in the real world, you may lose points because you are not working with the function you were given (only an approximation of it), and this is not one of the four allowed calculator operations. Do not curve fit.
- Using a built-in calculator utility or a program without showing all the work and justification for what you are doing. You may do only the four things mentioned above without further explanation.

- Finally, one common mistake on the exams is *not* using your calculator when you should. On the two calculator allowed sections if you need to compute a derivative at a point, evaluate a definite integral or solve an equation, you are expected to use your calculator even if you can do by hand. Use your calculator; don't waste your time.

Good Luck!