Solution Mean Numbers Question September 25, 2013

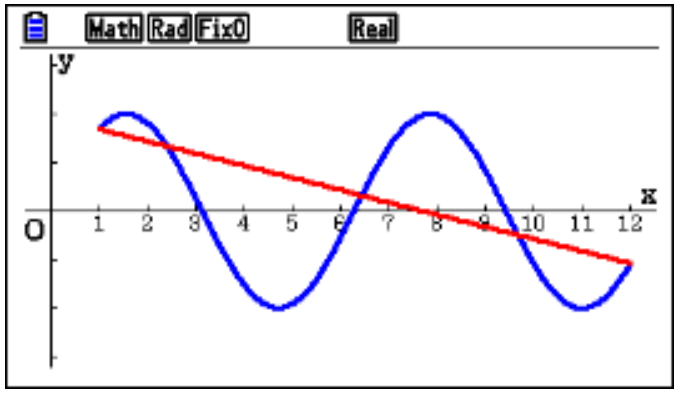
Consider the function $f(x)=\sin(x)$ on the closed interval [1, 12].

1. Write an equation of a line, $y(x)$ between the endpoints of the function.

   $y(x)=\sin(1)+\dfrac{\sin(12)-\sin(1)}{11}(x-1)$.

   $y(x)=-0.125277x+0.966748$

   Slope = –0.125277

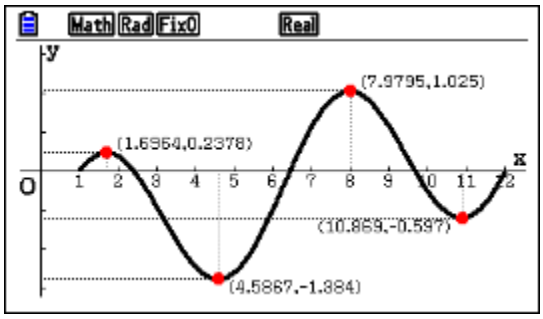



2. Write the equation of a function $h(x)$ that gives the vertical distance between $f(x)$ and $y(x)$. Since *f* may be both above and below *y* this function may have positive and negative values.

   $h(x)=f(x)-y(x)=\sin(x)-\left(\sin(1)+\dfrac{\sin(12)-\sin(1)}{11}(x-1)\right)$

   $h(x)=\sin(x)-012527x-0.96678$

3. Graph *h* and find its critical values. What are these places with respect to the graph of *h* and the graph of *f*.



Critical values: $x$ = 1.696403463, 4.586782052, 7.979588639, 10.86996724

These are the locations the local maximums and minimums of *h*. These are the locations of the places where the distance from *f* to *y* are locally longest or shortest.

4. Calculate the derivative of *f* at the critical values of *h*.

   $f'(x) = \cos(x)$

   cos(1.6964035) = – 0.125277
   cos(4.5867821) = – 0.125277
   cos(7.9795886) = – 0.125277
   cos(10.869967) = – 0.125277

   Note that – 0.125277 is the slope of $y(x)$

5. Interpret your result graphically.

   At these points the tangent lines to *f* are parallel to *y* the line between the endpoints. In other words these are the values of *c* guaranteed by the Mean Value Theorem.