Pacing for AP Calculus

Some suggestions by Lin McMullin.

The purpose of this note is to give you some guideline is planning your year's work for AP Calculus AB or BC. The ideas are my own and are only suggestions for you to consider.

Almost all textbooks provide an AP pacing guide among their ancillary material. You can consult the guide for your book for specific suggestions for the number of days on each topic or section.

Keep a copy of the latest Course Description ("The Acorn Book") handy. Changes in the exam are announced in this book; to keep up to date be sure you always read the following year's edition which is available at AP Central shortly after the exam is given in May. The book contains the "Topical Outline" for the AB and BC courses. The topics listed here are what may be tested on the exams. What is not listed will not be tested. For example, calculating volumes by the method of Cylindrical Shells is not listed; any volume problem on the exam can be done by other methods. This does not mean you may not or should not teach the topics that are not listed if you believe your students will benefit from them. If you wish to teach them you may still do so. Students may use these methods on the exam; they will not be penalized for correct mathematics. Many teachers teach these topics in the time after the exam.

PLANNING YOUR YEAR

Get out your school calendar. The AP Calculus exams are usually given during the first week in May; the exact date will be in the Course Description book and at AP Central.

- Count back about 2 school weeks from the exam date (don't count your spring break week). Allow an extra week if you are prone to many snow days. This time will be used for review. (This brings you to a week or so into April.)

- Count back two more weeks. I'll discuss what this should time should be used for later. (Mid-march) This is when you should aim to be done the material and ready to begin review. Finishing by the beginning of March is even better.

- Count the number of weeks between the beginning of school and the week above. (About 26 - 27 weeks if your start just after Labor Day; 28-30 weeks if you start in mid-August). This is the number of week you have to teach the material. Don't panic: the AB course is taught typically in college in 30 – 35 classes in one semester. You do have time, but by the same token, you still need to stick with the calendar and keep you students on it as well.

- Take half this number and find the middle week of the year. This is sometime in early to mid-December. To allow equal time for derivatives and integrals, this is when you should finish derivatives and start integration. Don't delay starting integration beyond the first class of the New Year.

- Now plan your work so that you can do it in the time allowed. You all want your students to do well. It is not unknown for teachers to spend a few extra days now and then to give extra work on derivative. But this time adds up. Remember half the exam is integration; you need to cover that too. Don't get behind.

Look over past exams. Learn what is tested and how it is tested and plan your time accordingly. Here are some hints as to where you can shave some time.

STARTING THE YEAR

- Summer assignments: Personally, I do not see the use in summer assignments. What is their purpose? To keep the material fresh in the kids' minds, I suppose. But the good students will do it right away and then forget anyway over the summer, and the others, will forget "everything" over the summer and try the assignment at the end of the summer and get nowhere.

- If you want to keep their minds on mathematics over the summer, assign a good book to read. Maybe they will spread that out over the summer. Reading suggestion: *Is God a Mathematician?* by Mario Livio.

- Ideally, limits and continuity should be taught in pre-calculus. Work with your pre-calculus teachers and help them arrange their curriculum so that the things students need to know coming into calculus are taught in pre-calculus. This is one of the things vertical teaming can accomplish. (Incidentally, be sure they do not start learning about derivatives and the slope of tangent lines in pre-calculus as some textbooks do; the time is better spent elsewhere.) Remember the delta-epsilon definition is not tested and is optional.

- DO NOT begin the year with a week or two (or even a day or two) of review of mathematics up to calculus. It won't help. Later in the year when you get to one of those topics students "should" know, they will have forgotten it all over again. So instead of a week or two (or more) of review at the beginning of the year, plan 10 – 15 minutes of review when these topics come up during the year. (You'll have to do this anyway.)

- If the first chapter of your textbook is review, as most are, skip this chapter. Make your first night's assignment to read this chapter and ask about anything they don't remember. This chapter can be used for reference when necessary later in the year.

- Do begin the year with derivatives (or limits and continuity if students have not studied this before). The very fact that this is new will help get and retain the students' interest.

DERIVATIVES

Here are some places you may shave a few days off while teaching derivatives:

- Computing derivatives is important. Product rule, Quotient rule, Chain rule are all tested on the exam. But look at some past exams: the questions are not that complicated. It is rare to find "monster" problems involving all three rules together along with radicals and trig functions.

Sure, give one or two of those, but the basics are what are tested. Furthermore, you can and should include these all thru the year, so students stay in practice.

- Optimization problems: Building a cheaper box or fencing in the largest field with a given amount of fence are great problems. They do not appear on the AP exams (at least not since 1982). They do not appear because the hard part is writing the model (the equation); if a student misses this they cannot earn anymore points in the problem. If these problems were on the exam, missing the equation means the student could not go on and cost the student all 9 points on a free-response question. Finding maximums and minimum, which require the same calculus thinking and techniques, are tested in other ways. On the multiple-choice section, optimization questions, if any, are of the easiest sort. The model may even be given, and there will be no more than one such question. Spend only a day or two on the modeling.

- Related Rate problems: These questions do appear on the exams. A multiple-choice question on related rates may appear. As with any multiple-choice question it cannot be too difficult. Every few years a related rate question shows as part of a free-response question. You cannot cut this out completely, but you can shave some time off here if you are short of time.

- Practice the differentiation skills, and later the antidifferentiation skills, and the concepts associated with derivatives by including them on all your tests. Make all tests cumulative from the beginning of the year; just a random question or two will keep them on their toes.

- Look for and assign differentiation problems based on graphs and tables of values in addition to the usual analytic (equation) questions. Use your textbook; however, some textbooks are rather thin on questions with tables and graphs in the stem. Use released exams or a review book for sources.

INTEGRALS

- As with derivatives, the finding of antiderivatives is important, but the antiderivatives, definite and indefinite integrals are not very difficult. There are no trig substitution integrals, and nothing too monstrous. Integration by Parts is only on the BC exam. Give students lots of practice spread over the second half of the year.

- Trapezoidal *Rule* is not really tested on the exams. Students do not need to know the formula or the error bound formula for the Trapezoidal Rule. Questions do ask for a "trapezoidal *approximation*." Like the left-, right; and midpoint-Riemann sums approximations, these questions can be answered by actually drawing a small number of trapezoids and computing their areas. This should be done from equations, graphs and tables. This tests the concept and often the graphical interpretation, not the mindless use of a formula. Error analysis is tested based on whether the approximating rectangles or trapezoids lie above or below the graph. Simpson's Rule is not tested.

- Look for and assign integration problems based on graphs and tables of values in addition to the usual analytic (equation) questions. Use your textbook, released exams or a review book for sources.

THOSE TWO WEEKS BEFORE THE REVIEW STARTS

The free-response and the multiple-choice sections of the exam contain some questions very similar to questions that are in textbooks and in contiguous sections of the textbook. These include:

- The area-volume question
- The differential equation question, and
- On the BC exam, the Power series question

The free-response and the multiple-choice sections contains some questions very different than questions that are in the textbooks. This is because these questions are on topics from different parts of the year (limit, differentiation and integration concepts in the same question), and these questions are just not asked in the same way in textbooks. These include:

- Rate/accumulation questions
- Functions defined by integrals
- Motion on a line (AB), or motion in a plane (BC – parametric and vector equations)
- Differentiation and integration questions about a function given the graph of its derivative
- Questions, both differentiation and integration, given a table of values.
- Overlapping topics in the same question such as a particle motion question based on a graph or table stem, or a question about an important theorem based on value in a table.

The topics in this latter list pull the entire year's work together. At first students find this disconcerting since they have rarely seen question like these; so be sure they do see them before the test. Use these two weeks to pull these topics together and get your students thinking more broadly. This will lead naturally into the full scale review; in fact some of this work may profitably spill over into the review time. Spend 2 – 3 days on each type using actual AP questions for each so the students can see the different variations on the same idea, and the different ways the same idea can be tested. (This is preferable starting the review with one complete free-response exam with 6 different type questions to do. However, later in the review you should do this.)

Another way to approach these problems is to include parts of them throughout the year as the students learn the topics tested in each part. Released multiple-choice problems can be used for this purpose as well.

THE REVIEW TIME

Once the students are familiar with the style of questions, give them a mock exam. For the multiple-choice questions use one of the released exams or one of the genuine-fake exams in a good review book. Give the free-response questions from a recent year. If possible give the mock exam under the same conditions and timing as the exam. This can be done on a Saturday. If you cannot get 3.25 hours in

a row, then give the parts with their proper timing during class periods. Grade the exam according to the standards which are available at AP Central. Teach them some good test taking strategies.

Spend a fair amount of time doing multiple-choice questions. The released exams from 1998, 2003 and 2008 are available. You can also use questions from a good review book. Pay attention to the style and wording, as well as the concepts tested.

Make your calendar up in advance and stick to it. You won't help the students by getting behind; in college they will have to go a lot faster than in high school. Help them get used to it.

I hope this helps you get started and keep a proper pace thru the year.